07-08 高等数学 A1 答案详解

一.填空题

$$1.\lim_{x\to 0}\frac{\sin 3x}{x}f\left(\frac{\sin 2x}{x}\right)=3f(2)=9$$

$$2.K=\frac{|y''|}{\left[1+(y')^2\right]^{2/3}}=\frac{2a}{\left[1+(y')^2\right]^{2/3}}$$ 最大，有$|y'|$最小，$y'=2ax+b=0\Rightarrow x=-\frac{b}{2a}$

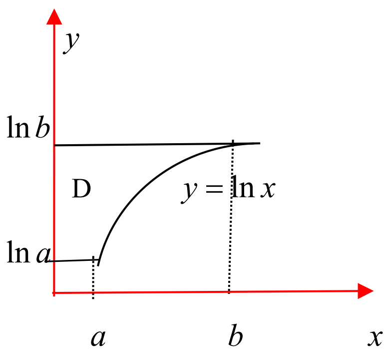
3.$\tan y=x+y$,两边取微分，有$\sec^2 ydy=dx+dy\Rightarrow dy=\cot^2 ydx$

$$4.\cos\langle a,b\rangle=\frac{a\cdot b}{|a||b|}=\frac{3-2+2}{\sqrt{9+1+4}\sqrt{1+1+4}}=\frac{3}{2\sqrt{21}}$$

二．单项选择

$$1.\lim_{x\to 0}(1+\cos x)^{\frac{3}{\cos x}}=2^3=8$$

$$2A=\int_{\ln a}^{\ln b}e^y dy=\int_0^a(\ln b-\ln a)dx+\int_a^b(\ln b-\ln x)dx$$



$$3.M=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{\sin x}{1+x^2}\cos^4 xdx=0(\text{奇函数在对称区间内积分为零})$$

$$.N=2\int_0^{\frac{\pi}{2}}\cos^4 xdx>0,P=-2\int_0^{\frac{\pi}{2}}\cos^4 x<0$$

故$M<N<P$

4.在$x=x_0$的某个邻域$U(x_0,\delta)$内泰勒展开,有$f(x)=f(x_0)+\frac{f'''(x_0)}{3!}(x-x_0)^3$

$f'(x)=f'''(x_0)(x-x_0)^2$,为一开口向下的抛物线,0为其极大值

$f'(x)=0\Rightarrow x=x_0$，但$f'(x)<0$故$x=x_0$不是极值点

$f''(x)=2f'''(x_0)(x-x_0)$易知左右两边变号，故$(x_0,f(x_0))$为拐点

三．解答下列各题

$$1.\lim_{h\to 0}\frac{f(a-h)-f(a+2h)}{h}=\lim_{h\to 0}-\frac{f(a-h)-f(a)}{-h}-2\lim_{h\to 0}\frac{f(a+2h)-f(a)}{2h}=-f'(a)+2f'(a)=f'(a)$$

$$2\lim_{x\to\frac{\pi}{2}}\left(x-\frac{\pi}{2}\right)\cot 2x\overset{u=x-\frac{\pi}{2}}{==}\lim_{u\to 0}\frac{u}{\tan 2u}=\frac{1}{2}$$

四．解答下列各题

$$1\int\frac{10^{2\arccos x}}{\sqrt{1-x^2}}dx=-\frac{1}{2}\int 10^{2\arccos x}d(2\arccos x)=-\frac{10^{2\arccos x}}{2\ln 10}+C$$

$$2.\int_0^2 f(x-1)dx \overset{x-1=u}{==} \int_{-1}^1 f(u)du=\int_{-1}^0\frac{1}{1+e^x}dx+\int_0^1\frac{1}{1+x}dx=\ln 2+\int_{-1}^0\frac{1+e^x-e^x}{1+e^x}dx$$
$$=\ln 2-1+\left[\ln\left(1+e^x\right)\right]_{-1}^0=2\ln 2-1+\ln\left(1+e^{-1}\right)$$

$$3.\int_0^{\frac{\pi}{2}}e^{2x}\sin 2xdx=\left[\frac{1}{2}e^{2x}\sin 2x\right]_0^{\frac{\pi}{2}}-\int_0^{\frac{\pi}{2}}e^{2x}\cos 2xdx=-\frac{1}{2}\left[e^{2x}\cos 2x\right]_0^{\frac{\pi}{2}}-\int e^{2x}\sin 2xdx$$
$$=\frac{1}{2}e^{\pi}+\frac{1}{2}-\int_0^{\frac{\pi}{2}}e^{2x}\sin 2xdx\Rightarrow\int_0^{\frac{\pi}{2}}e^{2x}\sin 2xdx=\frac{1}{4}\left(e^{\pi}+1\right)$$

五．应用题

$$1.x_t'\big|_{t=0}=2e^t=2,y_t'\big|_{t=0}=-e^{-t}=-1,x(0)=2,y(0)=1\quad K_{切线}=\frac{2}{-1}=-2.K_{法线}=\frac{1}{2}$$

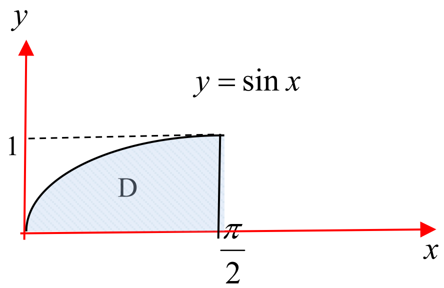
切线方程为$y-1=-2(x-2)\Rightarrow y=-2x+5$

法线方程为$y-1=\frac{1}{2}(x-2)\Rightarrow y=\frac{1}{2}x$

$$2.V=\frac{1}{3}\pi r^2h=\frac{1}{3}\pi h\left(20^2-h^2\right)\quad V'=\frac{400}{3}\pi-\pi h^2=0\Rightarrow h=\frac{20}{\sqrt{3}}$$

故$h=\frac{20}{\sqrt{3}}$时，体积最大

$$3.V=2\pi\int_0^{\frac{\pi}{2}}xydx=2\pi\int_0^{\frac{\pi}{2}}x\sin xdx=2\pi\left[-x\cos x+\sin x\right]_0^{\frac{\pi}{2}}=2\pi$$



六．证明题

假设极限存在且设为A,即$\lim\limits_{n\to\infty}x_n=A$

$x_n=\sqrt{2+x_n}$两边取极限,有$A=\sqrt{2+A}\Rightarrow A=2$

$$|x_n-2|=\left|\sqrt{2+x_{n-1}}-2\right|$$

$$=\left|\frac{x_{n-1}-2}{\sqrt{2+x_{n-1}}+2}\right|<\frac{1}{4}|x_{n-1}-2|<\frac{1}{4^2}|x_{n-2}-2|.....<\frac{1}{4^{n-1}}|x_1-2|\to 0(n\to 0)$$

即极限存在得证

七．解答题

$F'(x)=f(x)+\frac{1}{f(x)}\geq 2>0$　故$F(x)$单调递增

$(2)F(b)=\int_a^b f(t)dt>0, F(a)=\int_b^a \frac{1}{f(t)}dt<0$

有$F(b)F(a)<0$,又$F(x)$单调递增,故只有一根

八．解答题

$e^x(f(x)+f'(x))=0\Rightarrow\left[e^x f(x)\right]'=0\Rightarrow e^x f(x)=C$

又$f(0)=1\Rightarrow C=1$.故$f(x)=e^{-x}$

08-09 高等数学 A1 答案详解

一．单项选择题

1. $\lim\limits_{x\to 0}\left(x\sin\frac{1}{x}-\frac{1}{x}\sin x\right)=0-1=-1$

2. $y'=f'(\ln x)\frac{1}{x}\Rightarrow dy=\frac{f'(\ln x)}{x}dx$

3. $\int(e^x-3\cos x)dx=e^x-3\sin x+C$

4. $\int_{-a}^{a}(x^3+\sin^3 x)dx=0$(奇函数)

二．单项选择题

1. $\lim\limits_{x\to 0}\frac{f(x)-f(0)}{x}=f'(0)=2\Rightarrow$同阶无穷小

2. $y(-x)=-y(x)$故为奇函数

3. $\overrightarrow{NM}=(3,-4,5)\ \overrightarrow{NP}=(-1,-2,2)$. $\cos\angle MNP=\frac{\overrightarrow{NM}\cdot\overrightarrow{NP}}{\left|\overrightarrow{NM}\right|\left|\overrightarrow{NP}\right|}=\frac{-3+8+10}{\sqrt{50\times 9}}=\frac{\sqrt{2}}{2}\therefore\angle MNP=\frac{\pi}{4}$

4. $f'''(x)>0, f''(0)=0\Rightarrow f''(x)>0\Rightarrow f'(x)$单调递增

故$f(x)$单调递增,有$f'(0)<\frac{f(1)-f(0)}{1-0}=f'(\xi)<f'(1).\xi\in(0,1)$

5. 设$\int_0^1 f(x)dx=A\Rightarrow f(x)=x+2A$两边在$(0,1)$上积分,有　$A=\frac{1}{2}+2A\Rightarrow A=-\frac{1}{2}$

三．计算题

1. $\lim\limits_{x\to 0}\frac{\tan x-\sin x}{\sin^3 x}=\lim\limits_{x\to 0}\frac{\sin x(1-\cos x)}{\cos x\sin^3 x}=\lim\limits_{x\to 0}\frac{x\left(\frac{1}{2}x^2\right)}{x^3}=\frac{1}{2}$

2. 略

$3. \lim_{x\to0^+}(\sin x)^x = \lim_{x\to0^+} e^{x\ln\sin x} = \lim_{x\to0^+} e^{\frac{\ln\sin x}{\frac{1}{x}}} \overset{\frac{\infty}{\infty}.\text{洛}}{===} \lim_{x\to0^+} e^{-\frac{x^2}{\tan x}} = e^0 = 1$

四．计算下列各题

$$1. x'=2t, y'=-\sin t. \frac{dy}{dx}=\frac{y'}{x'}=-\frac{\sin t}{2t}. \frac{d^2y}{dx^2}=\frac{d\left(\frac{dy}{dx}\right)}{dt}\frac{dt}{dx}=\frac{-\cos t2t+2\sin t}{4t^2 2t}=\frac{\sin t-t\cos t}{4t^3}$$

$$2. \int\frac{xe^{x^2}}{1-2e^{x^2}}dx=\frac{1}{2}\int\frac{d\left(e^{x^2}\right)}{1-2e^{x^2}}=-\frac{1}{4}\ln\left|1-2e^{x^2}\right|+C$$

$$3. \int_1^4\frac{\ln x}{\sqrt{x}}dx=2\sqrt{x}\ln x\Big|_1^4-\int_1^4\frac{2}{\sqrt{x}}dx=8\ln2-4\sqrt{x}\Big|_1^4=8\ln2-4$$

五．证明题

令$F(x)=\sin x+\tan x-2x. F'(x)=\cos x+\sec^2 x-2=\cos x+\tan^2 x-1$

再令$G(x)=\cos x-1+\tan^2 x. G'(x)=-\sin x+2\tan x\sec^2 x=\sin x\left(2\sec^3 x-1\right)$

$\because 0<\cos x<1. x\in\left(0,\frac{\pi}{2}\right)\therefore \sec x=\frac{1}{\cos x}>1.$故$2\sec^3 x-1>0$

$\Rightarrow G'(x)>0. G(0)=0.\Rightarrow G(x)>0,$即$f'(x)>0. f(0)=0.\therefore f(x)>0$

$\Rightarrow \sin x+\tan x>2x$

六．解答题

$$\Phi(x)=\int_\pi^x f(x)dx=\int_\pi^{-\infty}f(x)dx+\int_{-\infty}^x f(x)dx=-1+\int_{-\infty}^x f(x)dx$$

$$x<0. \Phi(x)=-1. 0<x<\pi, \Phi(x)=-1+\int_{-\infty}^x f(x)dx=-1+\int_0^x\frac{1}{2}\sin xdx=-1+\frac{1}{2}(-\cos x)\Big|_0^x=-\frac{1}{2}-\frac{1}{2}\cos x$$

$$x>\pi. \Phi(x)=-1+\int_{-\infty}^x f(x)dx=-1+\int_{-\infty}^0 0dx+\int_0^\pi\frac{1}{2}\sin xdx+\int_\pi^x 0dx=0$$

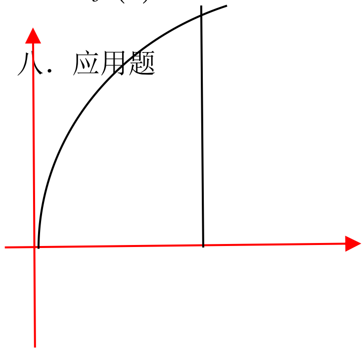
$$\therefore \Phi(x)=\begin{cases}-1 & ,x<0\\ -\frac{1}{2}-\frac{1}{2}\cos x\le x\le\pi & \\ 0 & ,x>\pi\end{cases}$$

七．解答题

$f(x)=\int_a^x f(x)dx\Rightarrow f(a)=\int_a^a f(x)dx=0$　原式两边求导. $f'(x)=f(x)\Rightarrow f(x)=Ce^x$

$x=a. f(a)=Ce^a=0\Rightarrow C=0\quad \therefore f(x)=\begin{cases}0, x=a\\ Ce^x, x\ne a\end{cases}$

八．应用题



$$S=\int_0^1 ax+bx^2dx=\frac{a}{2}+\frac{b}{3}=\frac{4}{9}\Rightarrow b=\frac{4}{3}-\frac{3}{2}a$$

$$V=\pi\int_0^1 y^2dx=\pi\left[\int_0^1\left(a^2x^2+\left(\frac{4}{3}-\frac{3}{2}a\right)^2x^4+2a\left(\frac{4}{3}-\frac{3}{2}a\right)x^3\right)\right]dx=\pi\left(\frac{1}{3}a^2+\frac{1}{5}\left(\frac{4}{3}-\frac{3}{2}a\right)^2+\frac{a}{2}\left(\frac{4}{3}-\frac{3}{2}a\right)\right)$$

$$V'=\pi\left(\frac{2}{3}a-\frac{3}{5}\left(\frac{4}{3}-\frac{3}{2}a\right)+\frac{1}{2}\left(\frac{4}{3}-\frac{3}{2}a\right)-\frac{3}{4}a\right)=\pi\left(\frac{2}{5}a-\frac{2}{15}\right)=0\Rightarrow a=\frac{1}{3}.b=\frac{4}{3}-\frac{3}{2}\frac{1}{3}=\frac{5}{6}$$

09-10 高等数学 A1 答案详解

一．单项选择题

$1.\lim\limits_{x\to 0}\frac{1-\cos x}{x^2}=1/2$(同阶不等价)

$2.\lim\limits_{x\to 0}x^2\sin\frac{1}{x}=0=f(0)$,故连续

$\lim\limits_{x\to 0}f'(0)=\lim\limits_{x\to 0}\frac{x^2\sin\frac{1}{x}-0}{x}=\lim\limits_{x\to 0}x\sin\frac{1}{x}=0=f'(0)$故可导

$3.\lim\limits_{x\to 1^-}f(x)=\lim\limits_{x\to 1^-}\frac{x^2-1}{x-1}=\lim\limits_{x\to 1^-}x+1=2$

$\lim\limits_{x\to 1^+}f(x)=2x=2$,故是可去间断点

$4.C$

二填空题

$1.\lim\limits_{x\to+\infty}\left(1-\frac{1}{x}\right)^{3x}=\lim\limits_{x\to+\infty}\left(1-\frac{1}{x}\right)^{-x\times(-3)}=e^{-3}$

$2.\lim\limits_{x\to 0}\frac{e^{3x\sin x}-1}{\tan x^2}=\lim\limits_{x\to 0}\frac{2x\sin x}{\tan x^2}=2$

$3.k=\lim\limits_{x\to\infty}\frac{y}{x}=\lim\limits_{x\to\infty}\frac{x}{3x+1}=\frac{1}{3}\qquad b=\lim\limits_{x\to\infty}(y-kx)=\lim\limits_{x\to\infty}\frac{-3x}{9x+3}=-\frac{1}{3}$

故斜渐近线为$y=\frac{1}{3}(x-1)$

$4.y'=-2\frac{\ln(1-x)}{1-x},\Rightarrow dy=2\frac{\ln(1-x)}{x-1}dx$

$5.y'=-xe^{-\frac{x^2}{2}}>0\Rightarrow x>0\quad y''=-e^{-\frac{x^2}{2}}+x^2e^{-\frac{x^2}{2}}<0\Rightarrow x^2<1\ \Rightarrow 0<x<1$

$6.\int xf''(x)dx=xf'(x)-\int f'(x)dx=xf'(x)-f(x)+C$

$7.\int_0^{+\infty}e^{-5x}dx=-\frac{1}{5}\left[e^{-5x}\right]_0^{+\infty}=\frac{1}{5}$

$8.\overline{y}=\frac{1}{\pi}\int_0^{\pi}\sin xdx=\frac{2}{\pi}$

$9.\int_{-1}^{1}(|x|+\sin x)x^2dx=2\int_0^1 x^3dx=\frac{1}{2}$

10.设$\int_0^1 f(x)dx=A.f(x)=x+2A$两边在$[0,1]$上积分，有$A=\frac{1}{2}+2A\Rightarrow A=-\frac{1}{2}$

三．计算下列各题

$1.x'=2t+1,y'=\cos t\quad \frac{dy}{dx}=\frac{y'}{x'}=\frac{\cos t}{2t+1},\frac{d^2y}{dx^2}=\frac{d\left(\frac{dy}{dx}\right)}{dt}\frac{dt}{dx}=\frac{-\sin t(2t+1)+2\cos t}{(2t+1)^3}$

$2.\sin(xy)+\ln(y-x)=x$两边取微分

$\cos(xy)(xdy+ydx)+\frac{dy-dx}{y-x}=dx,(0,1)$代入，解得

$dx+dy-dx=dx\Rightarrow\frac{dy}{dx}=1$ 故切线方程为$y=x+1$

四．计算题

$$1.\lim_{x\to 0}\frac{e^x-(1+2x)^{\frac{1}{2}}}{\ln(1+x^2)}=\lim_{x\to 0}\frac{e^x-e^{\frac{1}{2}\ln(1+2x)}}{x^2}=\lim_{x\to 0}\frac{e^x\left[1-e^{\frac{1}{2}\ln(1+2x)-x}\right]}{x^2}=\lim_{x\to 0}\frac{-\frac{1}{2}\ln(1+2x)-x}{x^2}$$

$$\overset{\frac{0}{0}洛}{=====}\lim_{x\to 0}\frac{-\frac{1}{1+2x}-1}{2x}=-\frac{1}{4}$$

$2.n\frac{n}{n+n^2}<A<n\frac{1}{1+n^2}$两边取极限，并由夹逼准则，得到 $\mathrm{A}=1$

五．解答下列各题

$0<x<3$时$,\Phi(x)=\int_0^x f(x)dx=\frac{1}{12}x^2$

$3\le x<4,\Phi(x)=\int_0^x f(x)dx=\int_0^3 f(x)dx+\int_3^x f(x)dx=\frac{3}{4}+\int_3^x 2-\frac{x}{2}dx=2x-3-\frac{1}{4}x^2$

$$\therefore\Phi(x)=\begin{cases}\frac{1}{12}x^2.0<x<3\\2x-3-\frac{1}{4}x^2.3\le x<4\end{cases}$$

$2.F(x)=x\int_0^x\arctan tdt-\int_0^x t\arctan tdt$

$F'(x)=\int_0^x\arctan tdt+x\arctan x-x\arctan x=\int_0^x\arctan tdt=x\arctan x-\int_0^x\frac{t}{1+t^2}dt$

$=x\arctan x-\frac{1}{2}\ln(1+x^2)$

$(2)F'(x)=\int_0^x\cos tdt=\sin x,F'\left(\frac{\pi}{2}\right)=1$

六．解答题

$$\frac{dy}{dx}\sin x = y\ln y \Rightarrow \frac{dy}{y\ln y} = \frac{dx}{\sin x} \Rightarrow \ln\ln y = \ln\left|\csc x - \cot x\right| + \ln c$$

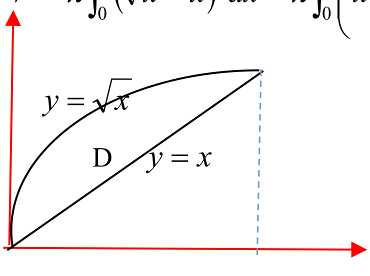
$$\Rightarrow \ln y = c(\csc x - \cot x)\text{又}y(\pi/2) = e,\text{则}c = 1 \therefore y = \csc x - \cot x$$

$$2.(x^2-1)y' + 2xy = \cos x,\text{即}\left[(x^2-1)y\right] = \cos x \Rightarrow (x^2-1)y = \sin x + C$$

七．应用题

$$A = \int_0^1\left(\sqrt{x} - x\right)dx = \left[\frac{2}{3}x^{\frac{3}{2}} - \frac{1}{2}x^2\right]_0^1 = \frac{1}{6}$$

$$V = \pi\int_0^1\left(\sqrt{x} - x\right)^2 dx = \pi\int_0^1\left(x + x^2 - x^{\frac{3}{2}}\right)dx = \frac{13\pi}{25}$$



八．证明题

(2)令$F(x) = f(x) - \sin x \quad F(0) = f(0) - 0 = 0, F(\pi/2) = f(\pi/2) - 1 = 0$

故由罗尔定理,至少存在一点$\xi \subset \left(0, \frac{\pi}{2}\right)$，使得

$F'(x) = f'(x) - \cos x = 0 \Rightarrow f'(x) = \cos x$

10-11 高等数学 A1 答案详解

一．单项选择题

$$1.\lim_{x\to 0}\frac{\sqrt{1+x} - \sqrt{1-x}}{x} = \lim_{x\to 0}\frac{(1+x)-(1-x)}{x\left(\sqrt{1+x} + \sqrt{1-x}\right)} = \frac{2x}{2x} = 1$$

2.D

3.令$F(x) = e^x - x - 2 \quad F'(x) = e^x - 1 > 0 \Rightarrow x > 0 \quad F(0) = -1, F(1) = e - 2 > 0$
故$(0,1)$内有一根，分析单调性可知，在定义域内也只有一个根

4.函数$f(x)$的所有原函数之间差一个常数,即$F(x) - G(x) = C$

二．填空题

$$1.\lim_{x\to 1^-} e^{\frac{1}{x-1}} = e^{-\infty} = 0$$

$$2.\lim_{x\to 0}\frac{\sin 3x}{x(1+\cos x)} = \lim_{x\to 0}\frac{3x}{2x} = \frac{3}{2}$$

3.两边取微分,有$e^y dy + ydx + xdy = 0 \Rightarrow dy = -\frac{y}{e^y + x}dx$

4.$\lim\limits_{x\to0} y = \lim\limits_{x\to0} xe^{\frac{1}{x^2}} = \lim\limits_{x\to0}\frac{e^{\frac{1}{x^2}}}{\frac{1}{x}} \overset{\frac{\infty}{\infty}.洛}{===} \lim\limits_{x\to0}\frac{-\frac{2}{x^3}e^{\frac{1}{x^2}}}{-\frac{1}{x^2}} = \lim\limits_{x\to0}\frac{2e^{\frac{1}{x^2}}}{x} = \infty$

故铅直渐近线为$x = 0$

5.$\int_0^1 xe^x dx = \left[xe^x - e^x\right]_0^1 = 1$

6.$\int_0^{+\infty}\frac{1}{1+x^2}dx = \arctan x\Big|_0^{+\infty} = \frac{\pi}{2}$

7.$I'(x) = (1-x^2)\arctan(x^2)(x^2)' = 2x(1-x^2)\arctan(x^2)$

8.$\bar{v} = \frac{1}{T_2 - T_1}\int_{T_1}^{T_2} v(t)dt$

三．解答下列各题

1(1)$\lim\limits_{x\to2}\frac{x^2+bx+a}{x^2-x-2} = \lim\limits_{x\to2}\frac{(x-2)(x+c)}{(x-2)(x+1)} = 2 \Rightarrow c = 4$

$(x-2)(x+4) = x^2 + 2x - 8 \Rightarrow a = -8, b = 2$

(2).由几何意义

$y = \frac{x^3+1}{x^2+1}$的斜渐近线与直线$y = -ax - b$平行且距离为1

那么$a = -\lim\limits_{x\to\infty}\frac{y}{x} = -\lim\limits_{x\to\infty}\frac{x^3+1}{x(1+x^2)} = -1$

$b = \lim\limits_{x\to\infty}(y - kx) - 1 = \lim\limits_{x\to\infty}\frac{1-x}{1+x^2} - 1 = -1$

2.$\lim\limits_{x\to0}\frac{\ln(1+x)-x}{x\ln(1+x)} \overset{\frac{0}{0}.洛}{===} \lim\limits_{x\to0}\frac{\frac{1}{1+x}-1}{\ln(1+x)+\frac{x}{1+x}} \overset{\frac{0}{0}.洛}{===} \lim\limits_{x\to0}\frac{-\frac{1}{(1+x)^2}}{\frac{1}{1+x}+\frac{1}{(1+x)^2}} = -\frac{1}{2}$

四．解答下列各题

1.$x' = \frac{1}{1+t^2}, y' = \frac{2t}{1+t^2} \quad \frac{dy}{dx} = \frac{y'}{x'} = 2t. \frac{d^2y}{dx^2} = \frac{d\left(\frac{dy}{dx}\right)}{dt}\frac{dt}{dx} = 2(1+t^2)$

3.$f'(x) = 3x^2 + 6x - 9 = 3(x-1)(x+3) > 0 \Rightarrow x > 1.x < -3$

故单调增区间为$(1,+\infty)$和$(-\infty,-3)$

单调减区间为$(-3,1)$ $\quad f''(x) = 6x + 6 = 0 \Rightarrow x = -1$

故凹区间为$(-1,+\infty)$,凸区间为$(-\infty,-1)$ $\quad$ 拐点为$(-1,15)$

五．解答下列各题

$1(1).\Rightarrow dy=\frac{\arctan x}{1+x^2}dx\Rightarrow y=\frac{1}{2}(\arctan x)^2+C$有$y|_{x=0}=1$,则$C=1\therefore 2y=(\arctan x)^2+2$

(2)根据题意,有$\frac{dy}{dx}=\frac{1}{\sqrt{9-x^2}}\Rightarrow y=\arcsin\frac{x}{3}+C$

又过$(0,1)\Rightarrow C=1$.所以曲线方程为$y=\arcsin\frac{x}{3}+1$

$2.\Rightarrow\frac{xy'-y}{x^2}=\cos x$.即$\left(\frac{y}{x}\right)'=\cos x\Rightarrow\frac{y}{x}=\sin x+C$

六．解答题

$-1<x<0.\Phi(x)=\int_{-1}^{x}2x+\frac{3}{2}x^2=x^2+\frac{1}{2}x^3-\frac{1}{2}$

$0\le x<1.\Phi(x)=\int_{-1}^{0}2x+\frac{3}{2}x^2dx+\int_0^x\frac{1}{1+e^x}dx=\frac{1}{2}+\int_0^x\frac{1+e^x-e^x}{1+e^x}dx=\frac{1}{2}+x-\ln(1+e^x)$

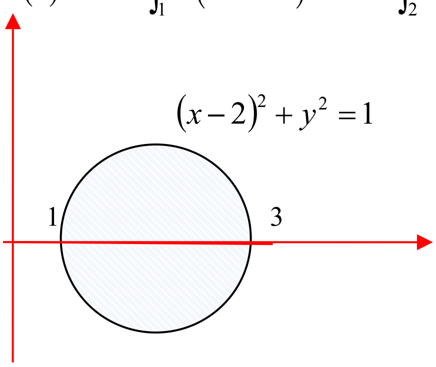
$$\therefore\Phi(x)=\begin{cases}x^2+\frac{1}{2}x^3-\frac{1}{2},-1<x<0\\ \frac{1}{2}+x-\ln(1+e^x),0\le x<1\end{cases}$$

七．应用题

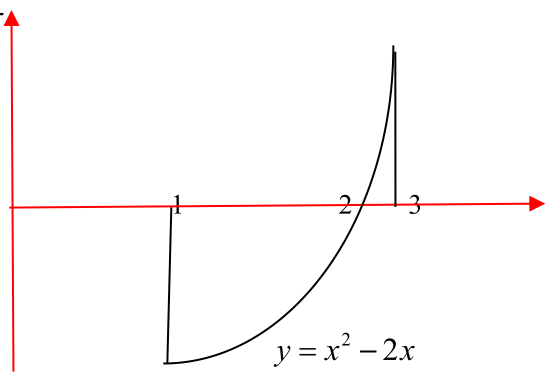
$(1)V=4\pi\int_1^3x\sqrt{1-(x-2)^2}dx\overset{x-2=\sin t}{==}4\pi\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(2+\sin t)\cos^2tdt=4\pi\int_0^{\frac{\pi}{2}}2\cos^2tdt=2\pi^2$

$(2)A=\int_0^2|y|dx=\int_0^2 2x-x^2dx=\frac{4}{3}$

$(2)V=2\pi\int_1^2x(2x-x^2)dx+2\pi\int_2^3x(x^2-2x)dx=9\pi$



七(1)图



七(2)图

八．证明题

$1(1)\int_0^2f(x)dx=\int_0^1f(x)-f(0)dx+\int_1^2f(x)-f(2)dx$

$\therefore\left|\int_0^2f(x)dx\right|=\left|\int_0^1f(x)-f(0)dx+\int_1^2f(x)-f(2)dx\right|=\left|\int_0^1f'(\xi_1)xdx+\int_1^2f'(\xi_2)(x-2)dx\right|$

$$\le\left|\int_0^1 f(\xi_1)xdx\right|+\left|\int_1^2 f(\xi_2)(2-x)dx\right|\le M\int_0^1 xdx+M\int_1^2(2-x)dx=\frac{1}{2}M+\frac{1}{2}M=M\left(\xi_1\in(0,1),\xi_2\in(1,2)\right)$$

$$\Rightarrow\left|\int_0^2 f(x)dx\right|\le M$$

(2)在(p,q)内,满足罗尔定理,即至少存在一点ξ_1,
使得$f'(\xi_1)=0,(\xi_1\in(p,q))$
同理,(q,r)内,也至少存在一点ξ_2,使得$f'(\xi_2)=0$
那么在(ξ_1,ξ_2)内也满足罗尔定理，至少存在一点$\xi\in(\xi_1,\xi_2)\subset(a,b)$
使得$f''(\xi)=0$

11-12 高等数学 A1 答案详解

一．单项选择题

1.极限存在与函数在改点是否有定义无关--D

$$2.\lim_{\Delta x\to 0}\frac{f^2(x+\Delta x)-f^2(x)}{\Delta x}=\left[f^2(x)\right]'=2f(x)f'(x)$$

$3.---B$

$4.F(x)=xf(x)+x^2$两边求导,有$f(x)=f(x)+xf'(x)+2x$

$\Rightarrow f'(x)=-2\Rightarrow f(x)=-2x+1$

二．填空题

$$1.\lim_{x\to 0}\left(\frac{1}{x}\sin x-x\sin\frac{1}{x}\right)=1-0=1$$

$$2.k=\lim_{x\to\infty}\frac{y}{x}=\lim_{x\to\infty}\frac{x}{2x+1}=\frac{1}{2}$$

$$3.f(x)=e^{\sin x\ln x},f'(x)=e^{\sin x\ln x}\left(\cos x\ln x+\sin x\frac{1}{x}\right)=x^{\sin x}\left(\cos x\ln x+\frac{\sin x}{x}\right)$$

$(1,2).(2.3)(3.4)$内均满足罗尔定理,至少存在$\xi_1.\xi_2.\xi_3$.使得$f(\xi_i)=0.i=1.2.3$

$f'(x)$为三次函数,最多有三个零点.故有3个零点

$$4.y'=\frac{1}{\sqrt{1-\sin x}}\frac{\cos x}{2\sqrt{\sin x}}\Rightarrow dy=\frac{\cos x}{2\sqrt{\sin x-\sin^2 x}}dx$$

5.根据e^x的泰勒展开,有$a_k=\frac{1}{k!}$

$$6.\int_1^{+\infty}\frac{1}{x^2}dx=-\frac{1}{x}\Big|_1^{+\infty}=1$$

7.0(被积函数为奇函数,积分区间对称.积分值为零)

8. $dA=\frac{1}{2}\rho^2(\theta)d\theta \qquad V=\int_c^d \pi\varphi^2(y)dy$

三．解答下列各题

1. $\lim\limits_{x\to\infty}\left(\frac{2x+1}{2x-1}\right)^{2x}=\lim\limits_{x\to\infty}\left(1+\frac{2}{2x-1}\right)^{2x}=\lim\limits_{x\to\infty}\left(1+\frac{2}{2x-1}\right)^{\frac{2x-1}{2}\frac{4x}{(2x-1)}}=e^{\frac{1}{2}}$

2. $n\frac{n}{n^2+n\pi}<A<n\frac{n}{n^2+\pi}$

两边取极限,并由夹逼准则可得$A=1$

3. $x'=1-\frac{1}{1+t}=\frac{t}{1+t},\ y'=3t^2+2t \quad \frac{dy}{dx}=\frac{y'}{x'}=\frac{(3t^2+2t)(1+t)}{t}=(3t+2)(1+t)=3t^2+5t+2$

$\frac{d^2y}{dx^2}=\frac{d\left(\frac{dy}{dx}\right)}{dt}\frac{dt}{dx}=\frac{(6t+5)(1+t)}{t}$

四．证明题

令$F(x)=2x-\ln\frac{1+x}{1-x}$

$F'(x)=2-\frac{1}{1+x}-\frac{1}{1-x}=2-\frac{2}{1-x^2}=\frac{-x^2}{1-x^2}<0$.在$(0,1)$上单调递减

$F(0)=0$.故$F(0)<0\Rightarrow 2x-\ln\frac{1+x}{1-x}<0\Rightarrow e^{2x}<\frac{1+x}{1-x}$

五.解答下列各题

1. $\int\ln\sin x\csc^2 xdx=-\ln\sin x\cot x+\int\cot^2 xdx=-\ln\sin x\cot x+\int(\csc^2 x-1)dx$
$=-\ln\sin x\cot x-\cot x-x+C$

$\int xe^{-2x}dx=-\frac{1}{2}xe^{-2x}+\frac{1}{2}\int e^{-2x}dx=-\frac{1}{2}xe^{-2x}-\frac{1}{4}e^{-2x}+C$

2. $\int\frac{x-2}{x^2+2x+3}dx=\frac{1}{2}\int\frac{2x+2-6}{x^2+2x+3}dx=\frac{1}{2}\int\frac{d(x^2+2x+3)}{x^2+2x+3}-3\int\frac{1}{(x+1)^2+2}dx$
$=\frac{1}{2}\ln(x^2+2x+3)-\frac{3}{\sqrt{2}}\arctan\frac{x+1}{2}+C$

3.令$\int_0^1 f(x)dx=A$ 两边在$(0,1)$上积分,有$A=\int_0^1\frac{1}{1+x^2}dx+A\int_0^1\frac{1}{\sqrt{1-x^2}}dx$

$\Rightarrow A=\frac{\pi}{4}+\frac{\pi}{2}A\Rightarrow A=\frac{\pi}{4-2\pi}$

六．解答题

$-1\le x\le 3, F(x)=\int_{-\infty}^{-1}f(t)dt+\int_{-1}^{x}f(t)dt=\int_{-1}^{x}(1-t^2)dt=\left(t-\frac{1}{3}t^3\right)_{-1}^{x}=x-\frac{1}{3}x^3+\frac{2}{3}$

$x>3, F(x)=\int_{-\infty}^{-1} f(t)dt+\int_{-1}^{3} f(t)dt+\int_{3}^{x} f(t)dt=\left(t-\frac{1}{3}t^3\right)_{-1}^{3}=-\frac{16}{3}$

$$\therefore F(x)=\begin{cases}0, x<-1\\ x-\frac{1}{3}x^3+\frac{2}{3}, -1\le x\le 3\\ -\frac{16}{3}, x>3\end{cases} \quad \lim_{x\to 3^-}F(x)=\left(3-\frac{1}{3}3^3+\frac{2}{3}\right)=-\frac{16}{3}=\lim_{x\to 3^+}F(x)=F(3)$$

$f(3^+)=0.f(3^-)=-8 \quad \therefore \mathrm{F}(x)$在$x=3$处连续不可导

七.应用题

$7.4x^2+y^2=4, 8x+2yy'=0\Rightarrow y'=-\frac{4x}{y}\therefore$ 切线方程为$Y-y=-\frac{4x}{y}(X-x)$

$X=0,$得$Y=\frac{4}{y}.Y=0.$得$X=\frac{1}{x}$

$S=\frac{1}{2}XY-\frac{1}{4}2\pi=\frac{2}{xy}-\frac{\pi}{2}=\frac{2}{x\sqrt{4-4x^2}}-\frac{\pi}{2}=\frac{1}{x\sqrt{1-x^2}}-\frac{\pi}{2}$

$S'=-\frac{2y+2xy'}{(xy)^2}=-2\frac{y-\frac{4x^2}{y}}{(xy)^2}=-2\frac{y^2-4x^2}{x^2y^3}=0\Rightarrow y=2x$代入$4x^2+y^2=4$得$x=\frac{1}{\sqrt{2}}$

$S_{\min}=2-\frac{\pi}{2}$

八．证明题

(二本)在(a,c)内满足拉格朗日中值定理,即至少存在一点$\xi_1\in(a,c)$，使得

$f'(\xi_1)=\frac{f(c)-f(a)}{c-a}>0$

同理,在(c,b)内至少存在一点$\xi_2\in(c,b)$,使得

$f'(\xi_2)=\frac{f(b)-f(c)}{b-c}<0$

则在(ξ_1,ξ_2)也满足拉格朗日中值定理,即至少存在一点$\xi\in(\xi_1,\xi_2)\subset(a,b)$

使得$f''(\xi)=\frac{f'(\xi_2)-f(\xi_1)}{\xi_2-\xi_1}<0$

(三本)因不恒为常数,故至少存在一点c,使得$f(c)>f(a)=f(b)$或$f(c)<f(a)=f(b)$

若$f(c)>f(a)=f(b)$.

则根据条件易得.在(a,c)上满足拉格朗日中值定理,即至少存在一点ξ.使得

$f'(\xi)=\frac{f(c)-f(a)}{c-a}>0$

同理.$f(c)<f(a)=f(b)$时,在(c,b)内至少存在一点ξ.使得

$$f'(\xi)=\frac{f(b)-f(c)}{b-c}>0$$

综上至少存在一点$\xi\in(a,b)$使得$f'(\xi)>0$

12-13 高等数学 A1 答案详解

一．单项选择题

1. $\lim\limits_{x\to 0^+}\frac{1}{x}=+\infty$,$\lim\limits_{x\to 0^-}\frac{1}{x}=-\infty$故极限$\lim\limits_{x\to 0}\frac{1}{x}$不存在$---D$

2. $\lim\limits_{x\to\infty}y=\lim\limits_{x\to\infty}\frac{1+e^{-2x^2}}{1-e^{-x^2}}=1--$故水平渐近线为$y=1$

$\lim\limits_{x\to 0}\frac{1+e^{-x^2}}{1-e^{-x^2}}=\frac{2}{0}=\infty---$故铅直渐近线为$x=0$

3. $---D$

4. $f'(\sin^2 x)=1-\cos^2 x\Rightarrow f'(x)=1-x$

$\Rightarrow f(x)=x-\frac{1}{2}x^2+C$

二．填空题

1. $\lim\limits_{x\to\infty}\left(\frac{x-1}{x}\right)^{2x}=\lim\limits_{x\to\infty}\left(1-\frac{1}{x}\right)^{-x\times(-2)}=e^{-2}$

2. $y'=\frac{1}{x}.y''=-\frac{1}{x^2}.y'''=(-1)^2\frac{2}{x^3}....y^{(n)}=(-1)^{n-1}\frac{(n-1)!}{x^n}$

3. 两边取微分,$xdy+ydx=e^{x+y}(dx+dy)$

$\Rightarrow xdy+ydx=xy(dx+dy)\Rightarrow dy=\frac{xy-y}{x-xy}dx$

4. $y'=e^{-x}-xe^{-x}.y''=-e^{-x}+xe^{-x}-e^{-x}=(2-x)e^{-x}=0\Rightarrow x=2$

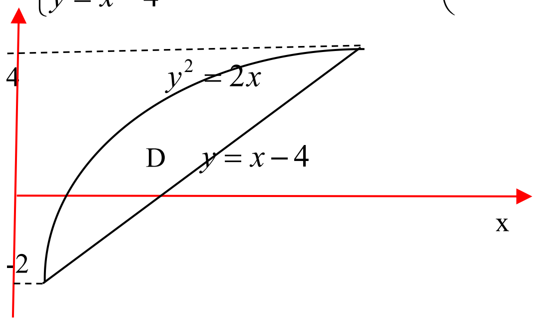
且在$x=0$边.y''异号.故拐点为$(2,2e^{-2})$

5. $\frac{d}{dx}\int_0^{x^2}\ln(1+t)dt=2x\ln(1+x^2)$

6. $\int_e^{+\infty}\frac{dx}{x\ln^2 x}=\int_e^{+\infty}\frac{d(\ln x)}{\ln^2 x}=-\frac{1}{\ln x}\Big|_e^{+\infty}=1$

7. $\rho(x_0)=\lim\limits_{x\to x_0}\frac{m(x)-m(x_0)}{x-x_0}=[m(x)]'\Big|_{x=x_0}=m'(x_0)$

8. $\begin{cases} y^2 = 2x \\ y = x-4 \end{cases} \Rightarrow (2,-2)(8,4) \therefore dA = \left(y+4-\frac{y^2}{2} \right) dy$



三．解答下列各题

$$1.\lim_{x\to 0}\frac{\sin 4x + x^2\sin\frac{1}{x}}{(1+\cos x)x} = \lim_{x\to 0}\frac{\sin 4x}{2x} = 2$$

$$2.\lim_{x\to 0}\frac{x-\arcsin x}{x^3} \overset{\arcsin x = t}{==} \lim_{t\to 0}\frac{\sin t - t}{\sin^3 t} \overset{\frac{0}{0}.\text{洛}}{====} \lim_{x\to 0}\frac{\cos t - 1}{3t^2} = \lim_{x\to 0}\frac{-\frac{1}{2}t^2}{3t^2} = -\frac{1}{6}$$

$$3. y = e^{x^2\ln\frac{x+2}{1-x}}. y' = e^{x^2\ln\frac{x+2}{1-x}}\left(2x\ln\frac{x+2}{1-x} + x^2\left(\frac{1}{x+2}+\frac{1}{1-x}\right)\right) = \left(\frac{2+x}{1-x}\right)^{x^2}\left(2x\ln\frac{2+x}{1-x} + \frac{3x^2}{(2+x)(1-x)}\right)$$

$$4. x' = 2t - \cos t + t\sin t + \cos t = t(2+\sin t). y' = 2 + \sin t$$

$$\frac{dy}{dx} = \frac{y'}{x'} = \frac{1}{t}, \frac{d^2y}{dx^2} = \frac{d\left(\frac{dy}{dx}\right)}{dt}\frac{dt}{dx} = -\frac{1}{2t^2(2+\sin t)}$$

四．解答下列各题

$$1.\int\frac{x+1}{x\sqrt{x-2}}dx = \int\frac{1}{\sqrt{x-2}}dx + \int\frac{dx}{x\sqrt{x-2}} = 2\sqrt{x-2} + \int\frac{dx}{(x-2+2)\sqrt{x-2}}$$

$$= 2\sqrt{x-2} + 2\int\frac{d\left(\sqrt{x-2}\right)}{2+\left(\sqrt{x-2}\right)^2} = 2\sqrt{x-2} + \sqrt{2}\arctan\frac{\sqrt{x-2}}{2} + C$$

$$2\int_0^{\frac{1}{2}} x\arcsin x dx \xrightarrow[dx=\cos u du]{\arcsin x = u, x=0, u=0, x=\frac{1}{2}, u=\frac{\pi}{6}} = \int_0^{\frac{\pi}{6}} u\sin u\cos u du = \frac{1}{2}\int_0^{\frac{\pi}{6}} u\sin 2u du$$

$$= \left(-\frac{1}{4}u\cos 2u + \frac{1}{8}\sin 2u\right)\Bigg|_0^{\frac{\pi}{6}} = \frac{3\sqrt{3}-\pi}{48}$$

五．解答题

$$0 < x \le 1. \Phi(x) = \int_0^x f(t)dt = \int_0^x e^t dt = e^x - 1$$

$$x > 1. \Phi(x) = \int_0^x f(t)dt = \int_0^1 e^x dx + \int_1^x \frac{1}{x}dx = e - 1 + \ln x$$

$$\therefore \Phi(x) = \begin{cases} e - 1 + \ln x, x > 1 \\ e^x - 1. 0 < x \le 1 \end{cases}$$

$\lim\limits_{x\to 1^+}\Phi(x)=e-1.\lim\limits_{x\to 1^-}\Phi(x)=e-1=\Phi(1)$故连续
$\lim\limits_{x\to 1^+}f(x)=1.\lim\limits_{x\to 1^-}f(x)=1$故不可导
$\therefore\Phi(x)$在$x=1$处连续不可导

六应用题

$$V=\pi\int_0^{2\pi}y^2(t)dx=8\pi\int_0^{2\pi}(1-\cos t)^3dt=8\pi\int_0^{2\pi}\left(2\sin^2\frac{t}{2}\right)^3dt=2^6\pi\int_0^{2\pi}\sin^6\frac{t}{2}dt$$
$$\overset{2u=t}{=\!=\!=}2^6\pi\int_0^{\pi}\sin^6 udu=2^7\pi\int_0^{\frac{\pi}{2}}\sin^6 udu=\frac{5}{6}\frac{3}{4}\frac{1}{2}\frac{\pi}{2}2^7\pi=20\pi^2$$

七.证明题

令$F(t)=f(t)-t$
$F(0)=f(0)>0.F(1)=f(1)-1<0$
有$F(0)F(1)<0$.则$F(t)$在$(0,1)$内有一点x.使$F(x)=0\Rightarrow f(x)=x$
又$F'(t)=f'(t)-1\neq 0$则必单调
所以只有一点满足$f(x)=x$

七．证明题

由积分中值定理有,$\frac{1}{b-a}\int_a^b f(x)dx=f(c)=f(b).c\in(a,b)$
则在(b,c)上满足罗尔定理,即至少存在一点$\xi\in(c,b)\subset(a,b)$
使得$f'(\xi)=0$

13-14 高等数学 A1 答案详解

一．单项选择题

1.$\lim\limits_{x\to 0}\sin x\sin\frac{1}{x}=0$.故是可去间断点

2.$---D$

3.$p=0.\int_0^1\frac{1}{x}dx=-\infty$不收敛

$p=2.\int_0^1\frac{dx}{x^{1-p}}dx=\int_0^1 xdx=\frac{1}{2}$收敛$---D$

4.0(定积分的值是个常数,而常数的导数为零)

二．填空题

$$1.\lim_{x\to 0}\frac{\cos x-1}{\left(1+x^2\right)^{\frac{1}{3}}-1}=\lim_{x\to 0}\frac{-\frac{1}{2}x^2}{\frac{1}{3}x^2}=-\frac{3}{2}$$

$$2.y'=\frac{1}{x}=2\Rightarrow x=\tfrac{1}{2}\Rightarrow \text{点}\left(\tfrac{1}{2},-\ln 2\right)\text{处的切线与}y=2x-3\text{平行}$$

$$3.\text{两边取微分,有}\sin xdy+y\cos xdx+\sin(x-y)(dx-dy)=0$$

$$\Rightarrow dy=\frac{y\cos x+\sin(x-y)}{\sin(x-y)-\sin x}dx$$

$$4.\int e^{1+\sin^2 x}\sin 2xdx=\int e^{1+\sin^2 x}d\left(1+\sin^2 x\right)=e^{1+\sin^2 x}+C$$

$$5.F(x)=\int_0^x f(t)dt=\int_0^3\left(1-x^2\right)dx+\int_3^x 0dx=x-\frac{1}{3}x^3\Big|_0^3=-6$$

$$6.V=\pi\int_a^b\left(f(x)-g(x)\right)^2dx$$

三．解答下列各题

$$1.\lim_{x\to+\infty}\left(1-\frac{1}{x^2}\right)^{3x}=\lim_{x\to+\infty}e^{3x\ln\left(1-\frac{1}{x^2}\right)}=\lim_{x\to+\infty}e^{-3x\times\frac{1}{x^2}}=e^0=1$$

$$2.x'=-e^t\sin t+e^t\cos t,y'=e^t(\sin t+\cos t)\quad \frac{dy}{dx}=\frac{y'}{x'}=\frac{\sin t+\cos t}{\cos t-\sin t}$$

$$\frac{d^2y}{dx^2}=\frac{d\left(\frac{dy}{dx}\right)}{dt}\frac{dt}{dx}=\frac{(\sin t-\cos t)^2+(\sin t+\cos t)^2}{(\cos t-\sin t)^2}\frac{1}{e^t(\cos t-\sin t)}=\frac{2}{e^t(\cos t-\sin t)^3}$$

$$3.\lim_{x\to+\infty}\frac{\int_1^x\left[t^2\left(e^{\frac{1}{t}}-1\right)-t\right]}{x}\overset{\frac{\infty}{\infty}.\text{洛}}{===}\lim_{x\to+\infty}x^2\left(e^{\frac{1}{x}}-1\right)-x\overset{u=\frac{1}{x}}{===}\lim_{u\to 0^+}\frac{e^u-1-u}{u^2}\overset{\frac{0}{0}.\text{洛}}{===}\lim_{u\to 0^+}\frac{e^u-1}{2u}=\frac{1}{2}$$

四．解答下列各题

$$1.\int_0^{\frac{\pi^2}{4}}\sin\sqrt{x}dx\overset{\sqrt{x}=t}{===}2\int_0^{\frac{\pi}{2}}t\sin tdt=-2t\cos t\Big|_0^{\frac{\pi}{2}}+2\int_0^{\frac{\pi}{2}}\sin tdt=\pi-2$$

五．解答题

$$\ln f(x)=3\ln(1+x)-2\ln(x-1)$$

$$\Rightarrow\frac{f'(x)}{f(x)}=\frac{3}{1+x}-\frac{2}{x-1}=\frac{x-5}{x^2-1}\Rightarrow f'(x)=\frac{(x+1)^2}{(x-1)^2}\frac{x-5}{x-1}<0\Rightarrow 1<x<5$$

$$f'(x)>0\Rightarrow x>5\text{和}x<1.f_{\text{极小}}(x)=f(5)$$

$$\ln f'(x)=2\ln(1+x)+\ln(x-5)-3\ln(x-1)$$

$$\frac{f''(x)}{f'(x)}=\frac{2}{1+x}+\frac{1}{x-5}-\frac{3}{x-1}=\frac{24}{(x+1)(x-5)(x-1)}\Rightarrow f''(x)=\frac{24}{(x+1)(x-5)(x-1)}\frac{(x+1)^2(x-5)}{(x-1)^3}$$

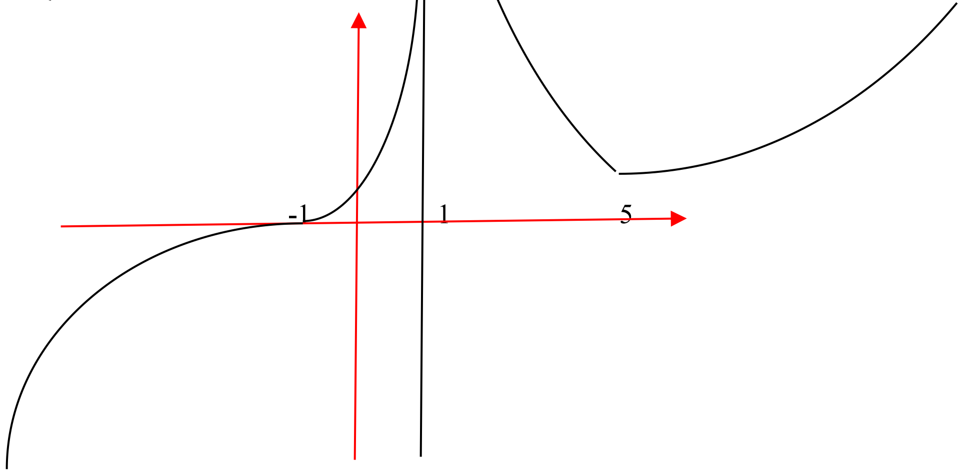
$$=\frac{24(x+1)}{(x-1)^4}>0\Rightarrow x>-1.f(-1)$$

x	(-无穷,-1)	-1	(-1,1)	1	(1,5)	5	(5,+ 无穷)
极 点 拐点		拐点				极小值	
单调，凹凸性	递增，凸		递增，凹		递减，凹		递增，凹

$$k=\lim_{x\to\infty}\frac{y}{x}=\lim_{x\to\infty}\frac{(x+1)^3}{(x-1)^2x}=1.b=\lim_{x\to\infty}(y-kx)=\lim_{x\to\infty}\frac{(x+1)^3-x(x-1)^2}{(x-1)^2}$$
$$=\lim_{x\to\infty}\frac{x^3+3x+3x^2+1-x^3+2x^2-x}{(x-1)^2}=5$$

$$\therefore \text{斜渐近线为} y=x+5.\ f(-1)=0.f(5)=\frac{6^3}{4^2}$$

$$\lim_{x\to-\infty}y=\lim_{x\to\infty}\frac{(x+1)^3}{(x-1)^2}=-\infty,\ \lim_{x\to+\infty}y=+\infty.\lim_{x\to1}y=\infty$$
故有铅直渐近线$x=1$



六．应用题

$$x'=-\sin t+\sin t+t\cos t=t\cos t,\ y'=\cos t-\cos t+t\sin t$$
$$s=\int_0^{2\pi}ds=\int_0^{2\pi}\sqrt{(x')^2+(y')^2}dt=\int_0^{2\pi}tdt=2\pi^2$$

$$2.\int\frac{2x-3}{x^2-2x+5}dx=\int\frac{2x-2-1}{x^2-2x+5}dx=\int\frac{d(x^2-2x+5)}{x^2-2x+5}-\int\frac{dx}{(x-1)^2+4}$$
$$=\ln(x^2-2x+5)+\frac{1}{2}\arctan\frac{x-1}{2}+C$$

七．证明题

因函数$f(x)$在闭区间$[c,d]$上连续.则必存在最大值M,跟最小值m

有$m\le f(c)\le M\Rightarrow \alpha m\le\alpha f(c)\le\alpha M$

$m \le f(d) \le M \Rightarrow \beta m \le \beta f(c) \le \beta M$
两式相加,有$m \le \alpha f(c) + \beta f(d) \le M$
由介值定理的推论,至少存在一点$\xi \in [c,d]$
使得$\alpha f(c) + \beta f(d) = f(\xi)$

八．证明题

令$F(x) = \frac{1}{a}\int_0^a f(x)dx - \int_0^1 f(x)dx = \frac{1}{a}\int_0^a f(x)dx - \int_0^a f(x)dx - \int_a^1 f(x)dx$

$\xrightarrow{\text{积分中值定理}} F(x) = f(\xi_1) - af(\xi_1) - (1-a)f(\xi_2) \quad (\xi_1 \subset (0,a), \xi_2 \subset (a,1))$

$F(x) = f(\xi_1) - f(\xi_2) + a(f(\xi_2) - f(\xi_1)) = (a-1)(f(\xi_2) - f(\xi_1))$

又$f(x)$连续单调递减，$\xi_1 \le \xi_2, f(\xi_2) - f(\xi_1) \le 0$，$a-1 \le 0$

$\therefore F(x) \ge 0 \Rightarrow \frac{1}{a}\int_0^a f(x)dx \ge \int_0^1 f(x)dx$